Constructing Condensed Memories in Functorial Time

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Abstract: If episodic memory is constructive, experienced time is also a construct. We develop an event-based formalism that replaces the traditional objective, agent-independent notion of time with a constructive, agent-dependent notion of time. We show how to make this agent-dependent time entropic and hence well-defined. We use sheaf-theoretic techniques to render agent-dependent time functorial and to construct episodic memories as sequences of observed and constructed events with well-defined limits that maximize the consistency of categorizations assigned to objects appearing in memories. We then develop a condensed formalism that represents episodic memories as pure constructs from single events. We formulate an empirical hypothesis that human episodic memory implements a particular time-symmetric constructive functor, and discuss possible experimental tests.

Keywords: condensed sets; pro-objects; frame problem; higher category theory; hypergraphs; sheaves

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1 Introduction

Time is a complex, still only partially formalized, notion that incorporates a number of distinct, not neccessarily mutually consistent meanings (Rovelli, 2021). In physics and computer science, time is typically formalized as "objective" in the sense of being observer or agent-independent; it can be considered to be defined by a designated "master clock" (e.g. the expansion of the universe) that cannot be perturbed by any agent. This global, shared, objective notion of time is familiar, in particular, from classical physics and non-relativistic quantum mechanics (e.g. Landau & Lifshitz, 1958). It underlies temporal logics in the lineage of A. N. Prior's (1957) tense logic **TL**, which have been broadly applied to formalize temporal reasoning in ordinary-language discourse about external objects and, particularly since Pnueli (1977), in execution traces generated by linear or concurrent computions (for reviews see Hodkinson & Reynolds, 2007; Fisher, 2008). "Time" in such logics may be discrete or continuous, linear or branching, but in either case it is unidirectional, i.e. representable as a directed acyclic graph (DAG). "Events" are effectively fragments of time slices of possible worlds (PWs), the states of which are viewed as time-dependent. They can be thought of as sets of time-labeled propositions that may, but do not necessarily, refer to past or future events, including past or future events on specific branches and at specific times. Time may be regarded as primitive, in which case events (or time-stamped states) are assigned to times and processes are driven by "next-time" operators that "jump forward" by a fixed interval (e.g. to the next second). Alternatively, events or states may be regarded as primitive, in which case times are assigned to events/states and processes are driven by "next-event/state" operators that are, effectively, next-time operators without a specified temporal interval (see Fisher, 2008, for discussion). Applications to asynchronous concurrent processes typically regard events/states as primitive; here the goal is often to show that undesirable outcomes such as deadlocks do not occur, i.e. that specified "safety" and "liveness" constraints are satisfied Lamport (1977); see Sistla (1994) for review. The internal "time" generated by event/state sequences within the system of interest is, in such cases, implicitly referenced to an external time in which the execution of the process is observed and its compliance with relevant constraints tested. Such observation and testing can be considered a two-player game between the system and some agent that observes and tests its behavior, in which a global, shared time remains unidirectional while process-execution time alternates between the players (Alur, Henzinger and Kupferman, 2002).

This objective notion of time can be contrasted with the subjective, "felt" or experienced time of some particular agent/observer. Our interest in the present paper is to develop a formal representation of this subjective, experienced time. We are interested, in particular, in experienced "long" times, e.g. the times between events recorded as episodic memories, as opposed to experienced "short" durations of particular events or event sequences (e.g. Matthews et al., 2014; Hoerl & McCormack, 2019; Roseboom et al., 2019). Here we follow Tulving (2002) in distinguishing between episodic (i.e. event) and seman-

tic (i.e. fact) memories, while acknowledging that this distinction is controversial (Klein, 2015) and that human episodic memory capabilities are widely variable Palombo (2018). We have two motivations for this focus on episodic memory. The first is the development in physics of observer-dependent, entropic definitions of time (Tegmark, 2012; Rovelli, 2017). Time is entropic if the future is more uncertain than the past; from the point of view of some observer, time is entropic if there is a net information flow inward from the observed environment, i.e. if learning exceeds forgetting. An entropic conception of time comports well with cosmological models in which spacetime is emergent from underlying quantum-informational processes (e.g. Swingle, 2012; Arkani-Hamed & Trnka J. , 2014; Pastawski *et al.*, 2015; D'Ariano & Perinotti, 2017; Bain, 2020). Nonobjective, though not necessarily entropic, conceptions of time can also be found in constructivist approaches to cybernetics (von Foerster, 2003; Fields *et al.*, 2017) and to the psychology of perception (Hoffman, Singh & Prakash, 2015; Fields *et al.*, 2018), and in some postmodernist philosophical thinking (Baudrillard, 1983; Deleuze & Guattari, 1987), and have a deep history in both Eastern (particularly Buddhist) and Western (anti-foundationalist and subjectivist) philosophy. Our second motivation is the increasingly well-established constructive nature of human episodic memory (Schacter & Addis, 2007; Hassabis & McGuire, 2009; Nadel *et al.*, 2012; Schwabe, Nader & Pruessner, 2014), which puts (retrospective) memory on a par with (prospective) planning as complementary forms of "mental time travel" (Boyer, 2008). Central to these capabilities are the inference of object persistence (Baillargeon, Spelke & Wasserman, 1985; Xu, 1999) and the ability to track object identities through (experienced) time (Scholl, 2007; Fields, 2016). These latter abilities are not human-specific, but characterize all animals able to recognize individual objects, e.g. objects used repeatedly as tools or particular conspecifics, at multiple times; however, it is not clear whether animals other than humans experience time itself in the way that humans do (Hoerl & McCormack, 2019). Our two motivations are not logically independent: identifying an object requires measurement, a physical process of information exchange that is enacted in spacetime (Fields, 2018a). When measurement is conceptualized in terms of subjective probabilities and Bayesian inference (e.g. Fuchs, 2010; Mermin, 2017; Fields, Friston, Glazebrook and Levin, 2022), measurements of time become agent-dependent and hence non-objective.

To develop a formal representation of nonobjective, experienced time, we follow Goguen (1991) by adopting a category-theoretic perspective. Category theory[1] provides a general

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[1]A *category* comprises a set of *objects* and a set of *morphisms* between objects, subject to the requirements that 1) every object has an associated Identity morphism and 2) morphisms compose associatively. A *functor* is a mapping between categories that respects Identity morphisms and morphism composition. Finite sets with functions, and groups, rings, fields, or topological spaces with their associated homomorphisms all form categories; indeed all of mathematics can be formulated in category-theoretic terms. For introductions to the theory, see e.g. Adámek, Herrlich and Strecker (1990), `https://en.wikipedia.org/wiki/Category_(mathematics)`, or the extensive resources of `https://ncatlab.org/`.

framework for mathematical modeling of arbitrary systems, and has been applied extensively in physics and computer science (see Fields & Glazebrook, 2019a, for examples). We employ three sets of mathematical tools. We begin by showing that events specified by finite numbers of n-ary relations on finite numbers of objects can be redescribed by finite multi-hypergraphs (MHGs) that generalize hypergraphs by allowing multiple distinct, labeled hyperedges connecting any subset of vertices. Events here correspond to "event files" as defined by Hommel (2004) in the short-duration limit; objects correspond to time-stamped "object tokens" as defined by Zimmer & Ecker (2010). These structures naturally give rise to an observer-relative, entropic conception of time. Time in this representation becomes both discrete and functorial, consistent with its functorial nature in topological field theories (Atiyah, 1988). We then reformulate events in the more expressive language of sheaves, and show how episodic memories, whether retrospective or prospective, can be viewed as intermediate steps between an experienced event and a sheaf-theoretic limit that captures the maximum information available in that event. Sheaf-theoretic methods have previously been employed in concurrency applications (Goguen, 1992; Sofronie-Stokkermans, 2009); we make similar use of sheaf-gluing conditions to enforce consistency conditions. As a final step, we employ the methods of Clausen & Scholze (2021) to condense episodic memories in this sheaf-theoretic representation onto a notional point interpretable as the present. This last step provides a model of episodic memory as fully constructive, and indeed provides a (formal) characterization of the construction process. It enables us to make a formal prediction that episodic memory is implemented by a particular, time-symmetric constructive functor in humans. We then examine some specific consequences of this prediction, including a critical role for stigmergic memories in rendering experienced time entropic, and consider experimental approaches that could test them.

2 Events as MHGs

Intuitively, an event occurs when something happens. One encounters a friend at a party, for example. Barwise and Perry (1983) define an "event" as a tuple $(Obj, Prop, Rel)$ where Obj is a finite set of individual objects $\{a_1 \ldots a_n\}$, $Prop$ is a set of unary properties $P(x)$, and Rel is a set of binary relations $R(x, y)$, where $x, y \in Obj$. Associated with each event is a collection of labels, including a label specifying a spacetime location. Such events are clearly amenable to organization with standard spatial and temporal logics that treat spacetime location as event-independent.

Somewhat broader concepts of "events" have been introduced by psychologists, beginning with the "event file" defined by Hommel (2004). An event file is a transient representation of objects, motions, and actions, as well as of affective states and motivations of agents including the self, the latter being information that would be attached as labels to an event as defined above. Event files capture an "instantaneous" situation, including occurent actions, in a short-duration limit of approximately 350 ms (Zmigrod & Hommel,

2011), but can also represent temporally-extended "events" when time is suitably coarse-grained (see also Altmann & Ekves, 2019; Cohn-Sheehy & Ranganath, 2017, for more recent extended event models). These psychologically-motivated notions of an "event" pertain, significantly, to the experiences of a single agent. When extended across sensory modalities, moreover, they characterize *all* that the agent experiences at a particular time. We can, therefore, add to the intuition that "something happens" in an event that what happens is experienced by some particular agent, and forms the complete experience, at that time, of that agent. This "completion" is attention-dependent and may be rather vague; the party at which one encounters one's friend may be crowded and noisy, with neither the crowd nor the noise resolved into further particulars.

Here we define an (complete, instantaneous, experienced) *event* as follows:

Definition 1. *An (complete, instantaneous, experienced)* event $\mathbb{V} = (\mathbb{A}, \mathbb{R}, \mathbb{L})$ *comprises a finite set $\mathbb{A}$ of n objects, a finite set $\mathbb{R}$ of unary to $(n-1)$-ary relations together with a unique n-ary relation V, and a finite list $\mathbb{L}$ of labels, each of which is a bit string of finite length.*

We interpret "object" broadly, e.g. considering the crowd in our example to be an object and "noisy($\cdot$)" to be a unary relation applying to it. The "membership" relation $V(a_1, \ldots a_n)$ indicates that the objects $a_1, \ldots a_n$ all occur in the single named event $\mathbb{V}$, which can be thought of informally as a component of an instantaneous state of a PW. As each event $\mathbb{V}$ has a unique membership relation V, we will use $V(a_1, \ldots a_n)$ as a short-hand for $\mathbb{V} = (\mathbb{A}, \mathbb{R}, \mathbb{L})$. We will also use the simplified notion $a, b, c, \ldots$ for objects and $P, Q, R, \ldots$ for relations within an event. The list $\mathbb{L}$ may include labels such as 'occurent percept' or '(episodic) memory' as discussed in §7 below. Definition 1 generalizes that of Barwise and Perry (1983) by allowing $(n-1)$-ary relations, and by not requiring the list $\mathbb{L}$ to include a label specifying a spacetime location. Spatial relations and hence spatial labels do not concern us here; temporal relations and hence temporal labels are constructed as discussed in §4 below.

We understand the above Definition 1 as referring to complete, instantaneous experiences of single agents. As such, events are neither decomposable nor composable. We explicitly do *not* assume any binary operations on events that yield events; an ordered pair $(\mathbb{V}_1, \mathbb{V}_2)$, for example, is not an event. This usage differs from agent-nonspecific notions of events in which, for example, simultaneous events in different components of a single distributed system are considered to be single "events" from the perspective of some 3rd party observer.

While the notation of 1st order logic is traditional for describing events, here we will employ the alternative notation of MHGs for reasons that will be come clear. Recall that a *graph* is a pair (X, E), where X is a set of *nodes* and E is a set of pairs (x, y), where $x, y \in X$, called *edges*. Note that this definition allows loops, i.e. edges (x, x). Assigning a unique label, a bit string of finite length, to each of the nodes, and similar but distinct

labels to each of the edges yields a *labeled graph*. A *labeled multigraph* is a pair (X, E^*), where X is a set of labeled nodes and E^* is a set of triples (x, y, l), where $x, y \in X$ and the label l is a bit string of finite length. Here any pair of nodes can be be connected by multiple edges that are distinguished by their labels. A *labeled, undirected hypergraph* is a pair (X, H), where X is a set of labeled nodes and H is a set of labeled subsets of X called *hyperedges*. By analogy with the multiple, labeled edges allowed between two nodes of a multigraph, we can define:

Definition 2. *A* labeled, undirected multi-hypergraph (MHG) *comprises a finite set X of m labeled nodes, and a finite set H^* of pairs (h, l_h), where h is a unary to m-ary hyperedge and its label l_h is a bit string of finite length.*

We will, for convenience, also refer to H^* as the "set of" hyperedges, leaving implicit the fact that each hyperedge has a distinct label. If a hyperedge $h = \{x_i\}, x_i \in X$, we will say that h *covers* each of the x_i.

   If two MHGs G and K each contain a node or hyperedge with some particular label, we will say that they *share* that node or hyperedge. By analogy with morphisms between graphs, we can define:

Definition 3. *Given MHGs G and K, an* MHG morphism *is a map $f : G \to K$, $f : X_G \mapsto X_K$ and $f : H_G^* \mapsto H_K^*$, where X_G, X_K and H_G^*, H_K^* are the nodes and hyperedges of G and K respectively, subject to the constraint that if $(\{x_i\}, l) \in H_G^*$ and $f : (\{x_i\}, l) \mapsto (\{y_j\}, f(l)) \in H_K^*$, then $f(x_i) \in \{y_j\}$ whenever $f(x_i)$ is defined.*

Note that the above definition allows an MHG morphism to be a partial function of either or both of X and H^*, i.e. to have a restricted domain in either set. Intuitively, an MHG morphism from G to K adds or deletes one or more nodes and/or hyperedges to/from G to produce K, leaving all shared node and hyperedge labels fixed; Fig. 1 shows two examples. Each MHG G has a unique associated MHG morphism $\mathrm{Id}_G : G \mapsto G$ that leaves the nodes and hyperedges of G fixed. Any three MHG morphisms $f : G \to K$, $g : K \to L$, and $h : L \to M$ obviously compose associatively. A category **MHG** can, therefore, be defined by taking MHGs to be the objects of **MHG** and MHG morphisms to be the morphisms of **MHG**.
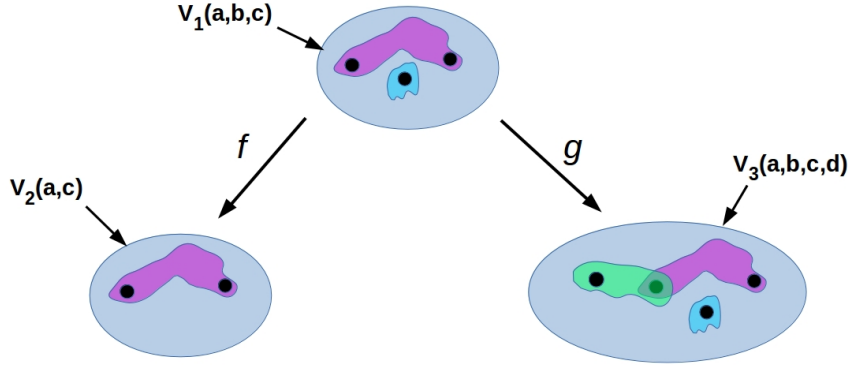
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Figure 1: Examples of MHG morphisms that delete (f) or add (g) objects (dark circles) and/or relations (colored shapes) to an MHG $V_1(a, b, c)$. These MHGs satisfy the requirements for being "events" and are labeled by their maximal hyperedges V_1, V_2, and V_3.

We will be particularly interested in MHGs in which the set X of nodes is interpreted as the set $\mathbb{A}$ of objects in some event $\mathbb{V}$, the set H^* of labeled hyperedges is interpreted as the set $\mathbb{R}$ of relations in $\mathbb{V}$, and there is only a single m-ary (i.e. maximal) hyperedge $V \in H^*$, where m is the number of nodes/objects in $\mathbb{V}$, interpreted as indicating that the objects $a_1, \ldots a_m$ co-occur in the single event $\mathbb{V}$. For simplicity, we will refer to MHGs with only a single m-ary hyperedge, i.e. MHGs amenable to this interpretation, as "events" and will consider the label l_V of the single m-ary hyperedge V to label or "name" the MHG as a whole. The subset of **Obj**(**MHG**) containing all and only such "event" MHGs can be rendered a subcategory **EVT** of **MHG** by requiring that if G and K are "event" MHGs, any MHG morphism $f : G \to K$ preserves the restriction to a single hyperedge covering all nodes; this can be done by requiring $f : V_G \mapsto V_K$ and $f : l_{V_G} \mapsto l_{V_K}$. We will refer to morphisms satisfying this restriction as "EVT morphisms" and employ the event notation $\mathbb{V} = (\mathbb{A}, \mathbb{R}, \mathbb{L})$ for MHGs in **Obj**(**EVT**). Note that, from an interpretative point of view, this restriction on EVT morphisms means that larger events subsume smaller ones, and hence enforces the idea that events are unitary entities that neither compose nor decompose into other, distinct events.

3   Object typing and object identity

Beginning in early infancy, humans segregate perceived objects from the "background" and assign them to types (or "cognitive categories"), e.g. as being a person, chair, tree, etc. based on their properties and relations (Baillargeon, Spelke & Wasserman, 1985; Xu, 1999). While nonhuman animals lack human-like grammatical languages, they exhibit a richness of communicative behavior that clearly indicates that specific objects are segregated from the general environment and assigned collections of properties or affordances, i.e. classified into types (Scott-Phillips& Heintz, 2022). The phylogenetic breadth and social learning of tool cultures and other group-specific behaviors similarly indicate well-developed object-typing and individual-object identification capabilities (Whiten, 2021). It is commonplace to treat such types as forming a strict hierarchy with a type name such as <thing> as the root; here and below we will use English words for type names as a convenience. Generalizing to a directed network with additional semantics is formally straightforward (Sowa, 1992) even if challenging in practice. Formally, we assume a finite, rooted DAG $Type$ in which each node i is labeled with a finite type name (e.g. <person> or <chair>) and each downward-directed edge $i \rightarrow j$ represents specialization to a less-inclusive (sub)type j. The leaf nodes of $Type$ can be considered to be labeled with names of minimally-inclusive subtypes that have at least two exemplars, i.e. singleton categories corresponding to particular individuals are not included in $Type$. Any sub-DAG of $Type$ with more than one node is, effectively, a taxonomy of its root type down to its leaf-node subtypes.

Definition 4. *A typing is an EVT morphism $C : (\mathbb{A}, \mathbb{R}, \mathbb{L}) \mapsto (\mathbb{A}, (\mathbb{R} \sqcup \mathbb{C}), \mathbb{L})$, where $\sqcup$ denotes disjoint union, that assigns a set $\mathbb{C}$ of "new" hyperedges to an event $(\mathbb{A}, \mathbb{R}, \mathbb{L})$.*

We employ the term "typing" to emphasize that assigning a type name to an object is a cognitive process that attaches further semantic information (the additional hyperedges in $\mathbb{C}$), beyond that specified by observed inter-object relations, to an event. We can represent an event with its attached typing as in Fig. 2.

Figure 2: a) A 4-object event $\mathbb{V}$, labeled by its membership relation $V(a, b, c, d)$, with 2-ary (orange) and 3-ary (magenta) relations between objects, "decorated" with unary and binary typings. b) The typings in a) can be regarded as "bundled together" (indicated by knotted string) over the event $\mathbb{V}$ to emphasize that they "add information" to $\mathbb{V}$. Note that any object can be covered by both one or more observed relations and one or more typings.

An "object token" as defined by Zimmer & Ecker (2010) represents a typed object within a given event. The 2-ary (orange) relation in Fig. 2 could, for example, be "next-to" and the 3-ary (magenta) relation could be "having the same velocity as"; these are spatial relations computed by the dorsal visual stream in humans and other mammals. In this case, the event depicted shows the instant at which one object is next to one of three co-moving objects; we can infer that one or the other of these relations does not hold in the immediately preceding or following events. Object typing depends not only on such spatial relations, but also on nonspatial information (e.g. size, shape, and color) computed by the ventral visual stream (see Goodale, 2014, for discussion); e.g. assigning the typing "is-a human" typically (although not necessarily) involves more than just location and motion information. *Identifying* an observed object as a specific individual, not just as a

member of some type, typically involves multiple type assignments, i.e. multiple labels from *Type*; identifying a specific individual person, for example, requires more information than recognizing an observed object as being a human, i.e. as being some person or other. The features, i.e. unary typings, of an object are often sufficient for individual identification, e.g. when recognizing a friend by her face. Beginning in childhood, humans also employ history information for individual identification (see e.g. discussion in Fields, 2012); we defer consideration of histories to §7.

4 Inter-event object identity induces temporal arrows

Now consider two distinct events $\mathbb{V}_1$ and $\mathbb{V}_2$ labeled by their membership relations $V_1(a, b, c)$ and $V_2(a, b, c, d)$, respectively, as shown in Fig. 3. The objects a, b, and c are present in both events, but what constitutes the *evidence* that a, for example, reappears in $\mathbb{V}_2$? This is the question of individual identification *in practice*, a question that remains unresolved despite decades of experimental work on humans and other animals, and despite millenia of in-principle philosophical speculation (Scholl, 2007; Fields, 2016)[2]. The only kind of evidence thus far defined is commonality of typing: here the object a has been assigned the same (possibly singular) typing in both $\mathbb{V}_1$ and $\mathbb{V}_2$, the object b has been assigned different typings in the two events, and the object c in $\mathbb{V}_1$ is joined by a new member, d of the same type in $\mathbb{V}_2$. A new person d, for example, may have walked into the room. We can consider these typings to indicate, effectively, "hypotheses" that a, b, and c are shared by $\mathbb{V}_1$ and $\mathbb{V}_2$, and that c and d are related by being of the same type but are non-identical. With this interpretation, we can view the typings as inducing a map:

$$T_{12} : a \mapsto a; b \mapsto b; c \mapsto c; \mathbb{V}_1 \mapsto \mathbb{V}_2$$

This map is clearly an EVT morphism. We will call this map T_{12} a "time" morphism from $\mathbb{V}_1$ to $\mathbb{V}_2$ as it captures the intuition that *time* is what connects events that share at least one object. In general, we can write a time morphism:

$$T_{ij} : \mathbb{V}_i \mapsto \mathbb{V}_j \tag{1}$$

connecting distinct events $\mathbb{V}_i$ and $\mathbb{V}_j$ that share at least one object. Directionality is imposed by requiring that T be a partial order on the set of all events, with the maps T_{ij} becoming arrows or compositions of arrows in the Hasse diagram for T. Directionality in this sense forbids temporal loops, but allows any event to have arbitrarily many "pasts" and "futures"; hence it fully captures the branching time constructs common in concurrency

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[2]We are assuming here that it is meaningful to talk about individuals being identifiable as such over time, an assumption that is questioned by multiple philosophical traditions. We do not engage here with this philosophical debate, but rather explore, in what follows, how observers go about identifying individuals, and how this process relates to the subjective experience of time.

applications. Note that past and future events may be isomorphic; no entropic or other means of distinguishing past from future events by content is assumed.
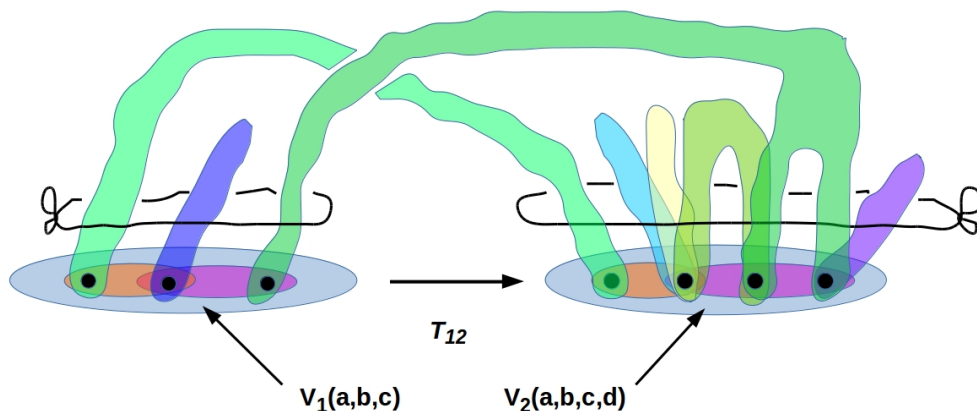

Figure 3: Two distinct events $\mathbb{V}_1$ and $\mathbb{V}_2$ labeled by their membership relations $V_1(a,b,c)$ and $V_2(a,b,c,d)$, respectively, joined by shared typings (light green and dark green shapes); T_{12} is the induced time morphism.

With this notation, we can characterize the **EVT** subcategory of **MHG** as the subcategory in which the objects are events with typings and the morphisms T_{ij} between distinct events are supplemented by identities that (abusing notation) can be written T_{ii}.

5 Entropic typing as retrospective time

To further investigate the relationship between typing and time, it is convenient to construct a functor:

$$\mathfrak{f} : \mathbf{Evt} \to \mathbf{Chu}$$

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from **Evt** to the category **Chu** of Chu spaces $(O, A, \models)$, where O is a set of objects, A is a set of attributes, and $\models\ \subseteq O \times A$ is a satisfaction relation (Barr (1979); see Fields & Glazebrook (2019a) for discussion and examples). Morphisms within **Chu** are pairs $\overrightarrow{f}, \overleftarrow{f}$ such that for Chu spaces $(O, A, \models)$ and $(O', A', \models')$:

$$\overrightarrow{f} : O \to O' \tag{2}$$

$$\overleftarrow{f} : A' \to A \tag{3}$$

such that:

$$\forall o \in O \text{ and } \forall a' \in A', \overrightarrow{f}(o) \models' a' \text{ iff } o \models \overleftarrow{f}(a'). \tag{4}$$

In the Channel Theory language of Barwise & Seligman (1997), the pair $\overrightarrow{f}, \overleftarrow{f}$ is an "infomorphism" between "classifiers" representable as Chu spaces[3].

Now consider events $\mathbb{V}_i$ with objects $\mathbb{A}_i$ and relations $\mathbb{R}_i$ and $\mathbb{V}_j$ with objects $\mathbb{A}_j$ and relations $\mathbb{R}_j$ such that:

$$\mathfrak{f} : \mathbb{A}_i \mapsto O; \mathbb{A}_j \mapsto O'; \mathbb{R}_i \mapsto A; \mathbb{R}_j \mapsto A',$$

for some Chu spaces $(O, A, \models)$ and $(O', A', \models')$, with $\mathfrak{f}$ the presumptive functor to be constructed. We can write $T_{ij} : \mathbb{V}_i \mapsto \mathbb{V}_j$ as the pair:

$$\overrightarrow{T_{ij}} : \mathbb{A}_i \to \mathbb{A}_j \tag{5}$$

$$\overleftarrow{T_{ij}} : \mathbb{R}_j \to \mathbb{R}_i \tag{6}$$

Consider now some object $o \in \mathbb{A}_i$ and some relation $r' \in \mathbb{R}_j$, and suppose that $\overrightarrow{T_{ij}} : o \mapsto o' \in \mathbb{A}_j$, i.e. that o is propagated forward from $\mathbb{V}_i$ to $\mathbb{V}_j$. The condition stated by Eq. (4) is satisfied if:

$$\forall o \in \mathbb{A}_i \text{ and } \forall r' \in \mathbb{R}_j, \overrightarrow{T_{ij}}(o) \text{ is covered by } r' \text{ iff } o \text{ is covered by } \overleftarrow{T_{ij}}(r'), \tag{7}$$

i.e. if *is covered by* functions as a Chu-space satisfaction relation. As a hyperedge is simply a set, this is clearly the case; hence the presumed $\mathfrak{f} : \textbf{Evt} \to \textbf{Chu}$ is indeed a functor.

Following the interpretation of the previous section, the forward component $\overrightarrow{T_{ij}}$ is the "time" through which objects evolve, while the backward component $\overleftarrow{T_{ij}}$ is the "time" of (retrospective, episodic) memory, restricted to require that the quantity of relational information – including typings – covering any object is preserved whenever the object is

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[3]Again see Fields & Glazebrook (2019a) for discussion and examples. Channel theory has been reformulated in the alternative language of *institutions* (Goguen & Burstall, 1992) in Goguen (2004).

preserved. Objects, in other words, do not lose information as they propagate through time. This restriction is effectively entropic: it requires $Card(\mathbb{R}_i) \leq Card(\mathbb{R}_j)$, which can be written as a "Second Law" $S(\mathbb{R}_i) \leq S(\mathbb{R}_j)$ by defining an "entropy" $S(\mathbb{X}) = \log(Card(\mathbb{X}))$ for any set $\mathbb{X}$. We can then define:

Definition 5. *An* entropic typing *is a typing of objects shared by events* $\mathbb{V}_i$ *and* $\mathbb{V}_j$ *for which* $\overrightarrow{T_{ij}}$ *is injective and* $\overleftarrow{T_{ij}}$ *is surjective on* $\mathbf{Dom}(\overrightarrow{T_{ij}})$.

In general, events involve varying numbers of objects and relations; hence a typing may fail to be entropic if it joins a current event involving few relations to a past event involving many relations. Using Def. 4 to write:

$$\mathbb{R} = \mathbb{R}^{core} \sqcup \mathbb{C}$$

We can see that the condition:

$$(S(\mathbb{C}_j) - S(\mathbb{C}_i)) > (S(\mathbb{R}_j^{core}) - S(\mathbb{R}_i^{core})) \tag{8}$$

guarantees increasing entropy from $\mathbb{V}_i$ to $\mathbb{V}_j$ and hence an entropic time operator T_{ij}. We can interpret Eq. (8) as requiring that entropic typings "build in past relational information" about the objects they type. One's friend may be typed, for example, as the kind of person who likes strong coffee and abstract ideas. This lays a foundation for viewing entropic typings as "histories" of objects in §7. Hence while such typings carry no explicit temporal information, they can be viewed as encoding expectations about how an object will behave in future events. Such expectations enable prospective episodic memory, i.e. prediction of and planning for future events. It is worth noting here explicitly that typings are an instance of *semantic* memory, and hence that typings being a key component of event representations is an instance of coupling between the semantic and episodic memory systems (see Renoult et al., 2019, for review).

6   Entropic typings as presheaves

We turn to the language of sheaves as a more expressive formalism with which to investigate the construction of temporal relations between events; indeed the basic intuition of a sheaf is suggested by the depicted typings being "tied together" in Figs. 2b and 3. This language allows us to construct temporal histories of individual objects, and to bind such histories together into episodic memories (Tulving, 2002) and generative models (Friston, 2010). We will see that such constructions can fail to be unique, producing a "plurivocity" of distinct temporal histories and hence "identities" for objects.

To begin, we note that by adding a null, i.e. 0-ary relation $\bar{\mathbb{V}}$ to each event, we can re-express events as discrete topological spaces in which every subset $\{a_k\} \in \mathbb{A}$ of objects related by some relation $R(a_k) \in \mathbb{R}^{core}$ is an open set. Adding this topological structure

allows us to move from the language of MHGs and their morphisms to the language of presheaves and then, in §8, the language of sheaves. Recall the idea of a presheaf:

Definition 6. (Hartshorne, 1977) *Let X be a topological space. A presheaf of sets on X is a contravariant functor* $\mathbf{F} : \mathbf{Op}(X) \to \mathbf{Sets}$ *on the category* $\mathbf{Op}(X)$ *of open sets of X.*

Intuitively, a presheaf assigns some set of data to each point, or neighborhood of points, on a topological space X. Since the presheaf operates on *all* open sets of X, the assignments must be self-consistent, i.e. larger neighborhoods inherit the assignments of data to their constitutive points. Taking X to be a set of events, we can regard typings as data assigned to the events. Hence we can state:

Lemma 1. *Entropic typings are presheaves on events.*

Proof. Let $\{\mathbb{V}_i\}$ be a set of of events with an entropic typing C. For any event $\mathbb{V}_i$, we consider a map $\mathbf{F} : \mathbb{A}_i \to \{\mathbb{C}_{ik}\}$; i.e. $\mathbf{F}$ maps each object to its type labels as assigned by $C(\mathbb{V}_i)$. Hence $\mathbf{F}$ inherits compositionality and respect for identities from C, so $\mathbf{F}$ is a functor. Contravariance of $\mathbf{F}$ is guaranteed by contravariance of $\overleftarrow{T_{ij}}$ whenever C is entropic. $\square$

We can think of the presheaf $\mathbf{F}$ as mapping each event $\mathbb{V}_i$ to its typing $C(\mathbb{V}_i)$; hence we will abuse notation slightly and write:

$$\mathbf{F} : \mathbb{V} \mapsto C \tag{9}$$

to indicate that C is an entropic typing of the set $\mathbb{V} = \{\mathbb{V}_i\}$ of events, viewed as a presheaf $\mathbf{F}$.

7   Constructed events and memories

Presheaves of events provide a natural representation for events as experienced. A key aspect of the construction of time, however, is the assumption that between any two distinct events experienced by some agent A, other things happened (i.e. non-agent-specific "events" occured) that A did not experience, but some other agent B may have experienced. This is especially the case during extended periods of non-observation. We all, for example, assume that various things happened between yesterday evening and this morning, and are eager to fill in knowledge of these happenings by checking the morning news. This "filling in" process is essential to the maintenance of object identity through time in a way that supports counterfactuals; an ability that requires temporal reasoning and may be human-specific (Hoerl & McCormack, 2019). This ability requires the generation of fictive (i.e. unobserved) causal histories (FCHs) that explain what objects were doing between

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observed events that include those objects (Fields, 2012, 2013). Such FCHs allow us to make immediate judgements about, for example, whether it is plausible that Jones, who we saw in Los Angeles on Saturday morning and are seeing again on Sunday afternoon, was in Paris on Saturday night.

From a concurrency perspective, an FCH constructed by an agent A is an *assumed* process $\mathcal{P}_a$ acting on some previously-observed object a, which is concurrent with the process $\mathcal{P}_A$ that propagates A (i.e. A's experienced "self") forward in time, and that satisfies the liveness condition that it delivers a into at least one future event experienced by A. The object a is, in this case, analogous to a datum that A needs in the future. The problem faced by A when experiencing the future event is determining whether the object that appears to be a is in fact a. This is a problem of provenance or chain of custody; a plausible FCH provides a heuristic solution (Fields, 2012). The solution is *only* heuristic, in that the satisfaction of relevant safety conditions – e.g. that a has not been modified in some critical but undetected way or replaced by an imposter – cannot in general be proven. The possibility that such safety conditions fail is obviously of importance in communication scenarios involving adverse actors; it also drives literary tropes of mistaken identity.

Consider now a set $\mathbb{V} = \{\mathbb{V}_i\}$ of observed events over which some entropic typing C defines a presheaf $\mathbf{F}$ via Lemma 1 above. To capture the action of FCHs between the events in $\mathbb{V}$, we need a way of "adding" events to $\mathbb{V}$ that respects all of the relational information between the $\mathbb{V}_i$. The concept of a "profinite set" accomplishes this:[4]

Definition 7. *A* profinite set *is a compact, Hausdorff, totally disconnected topological space that is a formal cofiltered limit of a collection of finite sets.*

The elements of a profinite set constructed as a limit of $\mathbb{V}$ "fill in" event-like elements "between" the observed events in $\mathbb{V}$ while maintaining the discrete topology; hence extending $\mathbb{V}$ to its profinite limit models the action of all possible relationship-preserving FCHs. An intuitive idea of what this means and how this "filling in" process works, is that the profinite limit is the "densest possible" set that includes the starting set and remains discrete. Call these filled-in elements *constructed* events $\widetilde{\mathbb{V}}_j$ and consider an entropic typing $\widetilde{C}$ over a profinite set $\widetilde{\mathbb{V}} = \mathbb{V} \sqcup \{\widetilde{\mathbb{V}}_j\} = \{\mathbb{V}_i\} \sqcup \{\widetilde{\mathbb{V}}_j\}$ such that:

$$
\begin{array}{ccc}
\widetilde{C} & \dashrightarrow & C \\
\big\uparrow{\scriptstyle \widetilde{\mathbf{F}}} & & \big\uparrow{\scriptstyle \mathbf{F}} \\
\widetilde{\mathbb{V}} & \xrightarrow{\;Proj\;} & \mathbb{V}
\end{array}
$$

commutes, where $Proj$ projects the observed events $\mathbb{V}$ out of the profinite limit $\widetilde{\mathbb{V}}$. The induced arrow in this case renders $\widetilde{C}$ the limit, over $\widetilde{\mathbb{V}}$, of the observed typing C. It "fills in"

---

[4]All standard definitions not otherwise referenced are from `https://ncatlab.org/nlab/show/HomePage`.

the appropriate type labels over the constructed extensions of the objects in the observed events $\mathbb{V}_i \in \mathbb{V}$, i.e. it creates their FCHs.

As a limit, $\widetilde{\mathbf{F}}$ is unique (up to isomorphism). In practice, we will be interested in a sequence of events that are "between" $\widetilde{\mathbb{V}}$ and $\mathbb{V}$ in terms of packing density, i.e. sequences that only partially "fill in" the "gaps" between the events in $\mathbb{V}$. Such sequences model only the FCHs that some agent actually constructs, not all possible FCHs. Hence we are interested in presheaves $\widetilde{\mathbf{F}}_k$ such that:

$$
\begin{array}{ccccc}
\widetilde{C} & \dashrightarrow & \widetilde{C_k} & \dashrightarrow & C \\
{\scriptstyle\widetilde{\mathbf{F}}}\big\uparrow & & {\scriptstyle\widetilde{\mathbf{F}_k}}\big\uparrow & & \big\uparrow{\scriptstyle\mathbf{F}} \\
\widetilde{\mathbb{V}} & \xrightarrow{Proj} & \widetilde{\mathbb{V}_k} & \xrightarrow{Proj} & \mathbb{V}
\end{array}
$$

(10)

commutes. Such a presheaf includes some, but not all, of the constructed events filled in to form $\widetilde{\mathbb{V}}$. The commutativity constraint can be expressed more succintly by requiring that for all k there are morphisms $\mathbf{G}_k, \mathbf{G}'_k$ such that:

$$
\widetilde{\mathbf{F}} \xrightarrow{\mathbf{G}_k} \widetilde{\mathbf{F}_k} \xrightarrow{\mathbf{G}'_k} \mathbf{F}
$$

(11)

These $\mathbf{G}_k, \mathbf{G}'_k$ are clearly associative and respect identity; hence they are functorial (we can also consider them as natural transformations of the relevant presheaves). We can, therefore, regard the nested presheaves $\widetilde{\mathbf{F}}, \ldots \widetilde{\mathbf{F}_k}, \ldots \mathbf{F}$ together with the functors $\mathbf{G}_k, \mathbf{G}'_k$ as forming a 2-category. The functors $\mathbf{G}_k, \mathbf{G}'_k$ pick out a particular intermediate presheaf $\widetilde{\mathbf{F}_k}$ that includes some, but not the maximal number, of constructed events with their associated entropic typings. We can think of $\mathbf{G}_k, \mathbf{G}'_k$ as implementing the process of constructing FCHs that connect observed events not to each other, but to the particular constructed event $\widetilde{\mathbf{F}_k}$.

The existence of $\mathbf{G}_k, \mathbf{G}'_k$ renders entropic time functorial: the local entropic time operators embedded in each presheaf $\widetilde{\mathbf{F}_k}$, and in the limits $\widetilde{\mathbf{F}}$ and $\mathbf{F}$, must associate and respect identities if the $\mathbf{G}_k, \mathbf{G}'_k$ do so. The sets $\widetilde{\mathbb{V}_k}$ of events, including the "observed" events $\mathbb{V}$ and the maximally "filled in" limit $\widetilde{\mathbb{V}}$ can, therefore, all be viewed as "small" categories. This categorical interpretation of the $\widetilde{\mathbb{V}_k}$ is natural given the neuroscience of "layers" of processing in which within-layer connections are interactions between representations of a given type, level of abstraction, and semantics, while "vertical" connections between layers are effectively maps between different types of representations at different levels of abstraction and with different semantics (see Fields & Glazebrook, 2019b, for extensive discussion).

Treating time as functorial relates the current, psychologically-motivated framework to relevant results obtained by physicists. The entropic condition expressed by Eq. (8) can be viewed as "classicalizing" the functorial time evolution of events represented by coordinate-free (hence "topological") quantum states (Atiyah, 1988), allowing a functorial

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time in a setting containing bounded and hence classical objects (cf. the construction of "objects" from quantum interactions in Fields, Glazebrook & Marcianò, 2021). This classicalization can be viewed as a coarse-graining, confirming the dependence of entropic time on coarse-grained "macroscopic" degrees of freedom emphasized by Rovelli (2019). To our view, these cross-disciplinary connections are not accidental, but rather speak to a deep relationship between experiences of time and of object identity. We return to this in §8 and 11 below.

Let us consider, as above, an entropic typing (i.e. presheaf) $\mathbf{F}$ of "observed" events. We are now in a position to state a formal definition of episodic memory as event memory:

Definition 8. *An* episodic memory *associated with an "observed" entropic typing* $\mathbf{F}$ *is a presheaf* $\widetilde{\mathbf{F}}_k$ *satisfying the commutativity constraint stated by Eqs.* (10) *and* (11).

An episodic memory is *retrospective* if its events are arranged in positive temporal order from some past event to the present, and is *prospective* if its events are arranged in positive temporal order from the present to some future event. A prospective episodic memory can also be considered a *plan* (see Schacter & Addis, 2007, for extensive discussion). We can also regard memories as either retrospective or prospective *histories* as this term is used in Fields (2012), bearing in mind that here "history" is as constructed by a remembering or planning agent, not "objective" in the sense of agent-independent.

We emphasize that between any two observed events $\mathbb{V}_1$ and $\mathbb{V}_N$ there can exist many distinct memories that may include both additional observed events and different numbers of filled-in constructed events. Distinct memories may encode different fictive causal histories of the objects appearing in $\mathbb{V}_1$ and $\mathbb{V}_N$ as illustrated in Fig. 4. These memories may impose inconsistent typings on the "boundary" events $\mathbb{V}_1$ and $\mathbb{V}_N$.
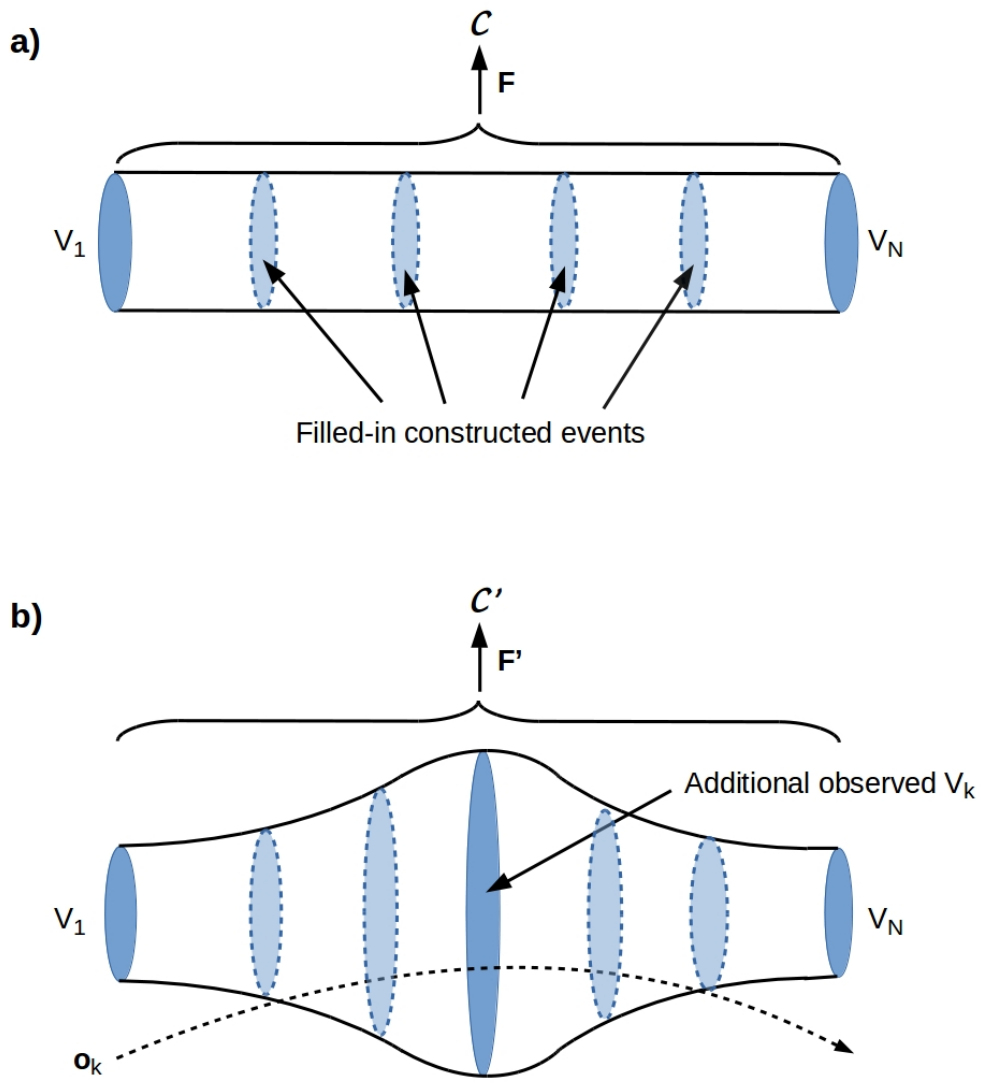
Figure 4: Two distinct memories connecting two observed events $\mathbb{V}_1$ and $\mathbb{V}_N$. Panel a) has only observed events $\mathbb{V}_1$ and $\mathbb{V}_N$, plus four constructed events (dashed ovals). Panel b) adds an observed event $\mathbb{V}_k$ in which an object o_k (dashed trajectory) not included in either $\mathbb{V}_1$ or $\mathbb{V}_N$ appears. Because they contain different sets of observed events ($\{\mathbb{V}_1, \mathbb{V}_N\}$ versus $\{\mathbb{V}_1, \mathbb{V}_k, \mathbb{V}_N\}$), they have different profinite limits that may, but may not, impose consistent typings.

Larger numbers of observed events further "classicalize" the functorial time evolution by imposing constraints on both the particular paths taken by the incorporated objects and, as illustrated in Fig. 4b, the co-occurrence of new objects and hence new potential interactions in particular "internal" events. Memories with large numbers of observed events are analogous, in a quantum-theoretic setting, to trajectories with intervening "which-path" measurements; however, we will not pursue this analogy here.

8   Memories as sheaves

The event-dependent, constructed time of retrospective and prospective memories provides, in the current framework, an alternative to the objective time of **TL** and its descendants. To explore some consequences of this non-objective representation of time, it is useful to complete the formal characterization of memories as sheaves. As noted in the Introduction, a sheaf-theoretic representation of event sequences provides a natural way to encode dependency relations between sequences in concurrency applications (Goguen, 1992; Sofronie-Stokkermans, 2009); here we will see that sheaves are equally useful for encoding logical relations between events and their contexts of observation. Sheaves are presheaves satisfying additional consistency constraints:

Definition 9. (Hartshorne, 1977) *Let X be a topological space. A sheaf $\mathcal{F}$ on X is a presheaf satisfying two axioms:*

- *Let U be an open subset of X and U_i an open cover of U. Given a collection of sections s_i on U_i, with $s_i|U_{ij} = s_j|U_{ij}$, then there exists a section s on U such that $s|U_i = s_i$.*

- *Let U be an open subset of X and U_i an open cover of U. If s is a section on U such that $\forall i$, $s|U_i = 0$, then s is zero.*

A sheaf is, in other words, a presheaf in which "nothing is missing" – all of the information that can be consistently assigned to the points in X has been assigned. In this regard, a sheaf $\mathcal{F}$ on a topological space X packs all the local data attached to open sets of X. As such, sheaves are tools used to transfer between local and global data: here data are global in the sense that some data are assigned to every open set of X, and local in the sense that data assigned on every open set can be restricted, in a compatible way, to data assigned on coverings of that open set, such that all of these data assignments are equivalent.

The collection $\{\mathbb{C}_k\}$ of type descriptors assigned by C to events $\mathbb{V}_i \in \mathbb{V}$ is clearly an open cover of the set $\mathbb{A} = \{\mathbb{A}_i\}$ containing any object that appears in any event contained in $\mathbb{V}$. Hence we immediately have:

Lemma 2. $\widetilde{\mathbf{F}}$ *is a sheaf on* $\widetilde{\mathbb{V}}$.

Proof. We consider the discrete topology on $\widetilde{\mathbb{V}}$. Both sheaf conditions are guaranteed by the functorial nature of the morphisms $\mathbf{G}_k, \mathbf{G}'_k$ between presheaves nested in $\widetilde{\mathbf{F}}$, i.e. by Eqs. (10) and (11). The entropic typing $\tilde{C}$ is, in particular, maximal because $\widetilde{\mathbb{V}}$ is profinite. $\square$

The fact that sheaves are "informationally complet" as outlined above allows us to "reverse the arrows" defined by $\mathbf{G}_k, \mathbf{G}'_k$. We use the following general construction:

Theorem 1. Sheafification (Theorem 1.2.9 of Alper (2021)) *Let S be a site and $\mathbf{Sh}(S)$ and $\mathbf{Pre}(S)$ be the categories of sheaves and presheaves, respectively, on S. The forgetful functor $\mathbf{f} : \mathbf{Sh}(S) \to \mathbf{Pre}(S)$ admits a left adjoint $\mathbf{f} \to \mathbf{f^{sh}}$, called the sheafification.*

Here we introduce Grothendieck's notion of a "site" as a way of making any category "look like" a topological space by putting a topology on a category. In particular, a site is a category endowed with a Grothendieck topology which constructs objects in the category to behave like open sets. Further, it abstracts the usual notion of a point to allow any object to "look like" a point in a topological space.

Recall from the discussion of Eq. (11) that the functors $\mathbf{G}_k, \mathbf{G}'_k$ render time functorial and hence the memories $\widetilde{\mathbb{V}_k}$ as well as their limits $\mathbb{V}$ and $\widetilde{\mathbb{V}}$ small categories. Hence $\widetilde{\mathbb{V}}$ can be considered a site. We have from Lemma 2 that $\widetilde{\mathbf{F}}$ is a sheaf on $\widetilde{\mathbb{V}}$. The functors $\mathbf{G}_k$ are clearly forgetful; hence Theorem 1 allows us to construct adjoints $\mathbf{G}_k^{sh}$ of the functors $\mathbf{G}_k$. In the present context, these adjoint functors effectively reverse the roles of objects and typings. The "upwards" construction in Fig. 2b has previously been shown (Fields & Glazebrook, 2019b) to have a "downwards" dual in which objects are viewed as labels ("instances") attached to the type descriptors, which now play the role of the "objects" being labeled. This duality has previously been explored in the setting of Barwise-Seligman classifiers and their associated infomorphisms. Here we consider this duality, for each episodic memory k, as the specific left adjoint $\mathbf{G}_k^{sh}$ defined above as the sheafification. Whereas $\mathbf{G}_k$ expresses a consistency condition on objects that is imposed by an entropic typing, the adjoint $\mathbf{G}_k^{sh}$ expresses a consistency condition on entropic typings that is imposed by (the assumption of) object identity. We can therefore dualize Eq. (11) as:

$$\widetilde{\mathbf{F}} \xleftarrow{\mathbf{G}_k^{sh}} \widetilde{\mathbf{F}}_k \xleftarrow{\mathbf{G}_k^{sh\prime}} \mathbf{F} \tag{12}$$

The $\mathbf{G}_k^{sh}, \mathbf{G}_k^{sh\prime}$ are effectively embeddings of memories within more-inclusive, but fully consistent, memories involving the same objects and typings, up to the limit specified by $\widetilde{\mathbf{F}}$.

The duality between objects and typings is useful, in that it illuminates the codependence between object identity and type identity, i.e. the persitence through time of types as coherent constructs. Recalling the discussion of Fig. 4b above, it is clear that this sheafification-induced duality depends on consistent typing at each step k of the embedding $\mathbf{G}_k^{sh}, \mathbf{G}_k^{sh\prime}$; Eq (4) can, therefore, be viewed as a test of consistency, i.e. of persistence,

through time. Failures of Eq (4) can be due to failures of object identity, e.g. an object losing some "essential" identifying property or relation in some incorporated event. "Mistakes" about object identity – particularly mistakes about the identities of other people – leading to unexpected consequences are common enough among humans to be a literary trope; see Scholl (2007) or Nichols & Bruno (2010) for examples and Fields (2012) for further discussion. They can also, however, be due to "intrinsic" (or "quantum") context changes, as discussed in this sheaf-theoretic context by Abramsky & Brandenburger (2011); see Fields & Glazebrook (2020) for further discussion. As a "context" in this sense is specified by a set of objects and relations, "enlarging" an event by embedding it in a larger event risks context change, and hence failure of Eq (4). Increasing the number of degrees of freedom of a joint system that are measured, for example, can introduce context shifts and hence violations of the Kolmogorov axioms by the joint distributions of observational outcomes in a quantum setting (Kochen & Specker, 1967); see Dzhafarov, Cervantes & Kujala (2017) for proof that such context shifts can be fully modeled in classical probability theory. Events that introduce new objects and relations, as illustrated in Fig. 4b, must in principle be proved to introduce no significant context change. The question of how to construct such proofs is known in AI as the Frame Problem (McCarthy & Hayes, 1969); it is now known to be intractable Dietrich & Fields (2020); again see Fields & Glazebrook (2020) for discussion in this context.

9   Sheaves over mutually-consistent memories

Consistency of typing across a collection of memories is effectively a gluing condition; hence we can re-express the consistency condition implicit in Eq. (12) through a further sheaf construction. We follow a procedure one of us recently used to conjecture a pro-diamond (Dobson, 2021a,b) towards a theory of pro-emergent time; here, we construct a pro-object of the category **Shv** of sheaves. Def. 7 can be generalized to:

Definition 10. *A pro-object of a category* **C** *is a formal cofiltered limit of objects of* **C**.

Note that a profinite set is a pro-object in **FinSet**. Indeed, pro-objects of any category **C** bear the relationship to the objects of **C** that profinite sets bear to finite sets: they are "completions at maximum density" of their objects. The category of pro-objects of an arbitrary category **C** is written **Pro**-**C**, and meets the following conditions:

- The objects are pro-objects in **C**.

- The set of arrows from a pro-object $F : D \to \mathbf{C}$ to a pro-object $G : E \to \mathbf{C}$ is the limit of the functor $(\mathbf{D^{op}} \times E) \to \mathbf{Set}$ given by $\mathbf{Hom_C}(F(\cdot), G(\cdot))$.

- Composition of arrows arises, given pro-objects $F : D_0 \to C$, $G : D_1 \to C$, and $H : D_2 \to C$ of C, by applying the limit functor for diagrams $(\mathbf{D^{op}} \times E) \to \mathbf{Set}$ to

the natural transformation of functors $(\mathbf{Hom_C}(F(\cdot), G(\cdot)) \times \mathbf{Hom_C}(G(\cdot), H(\cdot))) \to \mathbf{Hom_C}(F(-), H(-))$ given by composition in $\mathbf{C}$.

- The identity arrow on a pro-object $F : D \to C$ arises, using the universal property of a limit, from the identity arrow $\mathbf{Hom_C}(F(c), F(c))$ for every object c of $\mathbf{C}$.

This construct allows us to tighten the formal relationship between typings and the objects and events that they characterize. This is expressed by the following:

Theorem 2. *The sheaf over entropic typings of objects/events is a projective limit of a sheaf over objects/events. Therefore, the sheaf over typings is a pro-object in* $\mathbf{Shv}$*, the category of sheaves.*
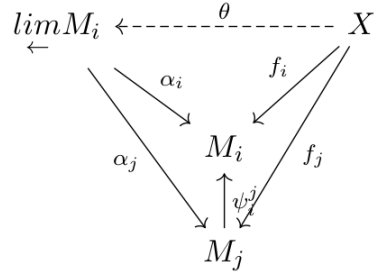
Proof. Let I be a partially ordered set. Recall the following:

Definition 11. Rotman (2000) *Given a partially ordered set I and a category $\mathbf{C}$, an inverse system in $\mathbf{C}$ is an ordered pair $((M_i)_{i \in I}, (\psi_i^j)_{j \geq i})$ abbreviated $\{M_i, \psi_i^j\}$, where $(M_i)_{i \in I}$ is an indexed family of objects in $\mathbf{C}$ and $(\psi_i^j : M_j \to M_i)_{j \geq i}$ is an indexed family of morphisms for which $\psi_i^j = 1_{M_i}$ for all i, and such that the following diagram commutes whenever $k \geq j \geq i$.*

$$
\begin{array}{ccc}
M_k & \xrightarrow{\ \ \psi_i^k\ \ } & M_i \\
& \searrow_{\psi_j^k} \quad \nearrow_{\psi_i^j} & \\
& M_j &
\end{array}
$$

Now let M_i be the graded sheaf over i-objects/events, $\mathbf{Shv}$ the category of sheaves, and $\{M_i, \psi_i^j\}$ an inverse system in $\mathbf{Shv}$ over I. Take the sheaf over typings of objects/events as an object $\underleftarrow{\lim} M_i$. By definition of entropic typing we have a family of projections $(\alpha_i : \underleftarrow{\lim} M_i \to M_i)_{i \in I}$. For our inverse system to be a projective limit we need:

- i) $\psi_i^j \alpha_j = \alpha_i$ for $i \leq j$,

- ii) for every $X \in obj(\mathbf{Shv})$ and all morphisms $f_i : X \to M_i$ satisfying $\psi_i^j f_j = f_i$ for all $i \leq j$, there exists a unique morphism $\theta : X \to \underleftarrow{\lim} M_i$ making the diagram commute.

$$\varprojlim M_i \xleftarrow{\quad\theta\quad} X$$

with arrows α_i, f_i, α_j, f_j into M_i, and ψ_i^j between M_i and M_j.

Conditions i) and ii) are just the consistency conditions for sequentially embedding the entropic typings $\widetilde{C_k}$; they are met whenever (12) is satisfied as discussed above. $\qquad\square$

Intuitively, Theorem 2 constructs a maximal consistent episodic memory $\mathcal{M}$ of the event M_i as recalled at the event M_j. Hence it provides a representation of the "best possible" memory between any two observed events. As noted above, there may be other memories, inconsistent with $\mathcal{M}$ and at least some of its components, that connect these events and include object identity changes or context shifts. These can be considered "failures" or "pathologies" of – or simply as design flaws in – the memory system as discussed further in §11 below.

10 Condensing memories onto the present

Any account of episodic memory must eventually face questions of implementation, both for the memory itself as a data structure and for the recall mechanism that retrieves one or more memories in response to some cue. The "picture" of memories as sheaves, or as in Theorem 2, projective limits of sheaves, at least suggests the traditional (or "reconstructive" or "preservationist") view of episodic memories as explicit records (or "traces") stored in a "library" of sorts and recalled via some kind of indexing system Robins (2017). This explicit view of episodic memory takes the common phenomenology of "clear and distinct" memories of actual past events at face value, considering them "ground truth" points between which FCHs may be interpolated. It is the model of memory most straight-forwardly implemented in symbolic (i.e. "GOFAI") AI systems, where arbitrarily-detailed event memories can be written to stable declarative data structures, e.g. images or text files.

The explicit view of episodic memory has been challenged by empirical results supporting a constructive view (Schacter & Addis, 2007; Hassabis & McGuire, 2009; Nadel *et al.*, 2012; Schwabe, Nader & Pruessner, 2014) in which even observed events are constructed "on the fly" and in a current-context dependent way (see Addis, 2018; Werning, 2020; Perrin, 2021, for recent reviews). This constructive view suggests that what is remembered is not a set of explicitly-represented events, but rather a set of operators with

which to construct representations of such events. The previously-firm distinction between "observed" and "constructed" events thus drops away; all episodic memory becomes FCH construction, i.e. more or less constrained imaginative confabulation. How the construction process is constrained, and the conditions under which these constraints fail, thus become outstanding empirical questions.

Here we employ methods developed by Scholze (2017) and Clausen & Scholze (2021) to re-express the previous sheaf-theoretic "picture" of episodic memory in terms of a "condensed" object located at a single notional "point" that we interpret informally as "the present" without committing ourselves to any particular position in the presentism–eternalism debate (e.g. Noonan, 2013; Rovelli, 2019). This condensed object can, in turn, be considered a representable functor, and so effectively a family of operators invokable at the present. These operators construct extended, multi-event representations of the (retrospective) past or the (prospective) future. This operator-based "picture" of episodic memory is purely constructive: recalling a past event or planning a future event consists in executing one or more such families of operators. Hence while the formal construction that follows is somewhat complicated, its interpretation is simple and, from a biological or cognitive-science point of view, fairly obvious.

Let $\mathbf{C}$ be a category, and let $\mathbf{Cond}(\mathbf{C})$ denote the category of "condensed" objects of $\mathbf{C}$. Clausen & Scholze (2021) show that $\mathbf{Cond}(\mathbf{C})$ can be represented as the category of small sheaves on $\mathbf{C}$, or equivalently as a representable functor $\mathbf{F} : \mathbf{C^{op}} \to \mathbf{Set}$. More formally, we have:

Definition 12. (Clausen & Scholze (2021) Definition 1.2) *The* pro-étale site $*_{proét}$ *of a point is the category of profinite sets* **Pro-FinSet**, *with finite jointly surjective families of maps as covers. A* condensed set *is a sheaf of sets on* $*_{proét}$. *Similarly, a condensed ring/group/object is a sheaf of rings/groups/objects on* $*_{proét}$.

We can understand this definition as follows. Condensed sets capture similar phenomena as topological spaces, but, categorically speaking, condensed sets formally behave like sets. A condensed set X measures the mapping of profinite sets S into X in the following way, as described by Scholze himself:

> Let me describe what a condensed set X "is": For each profinite set S, it gives a set $X(S)$, which should be thought of as the "[continuous] maps from S to X", so it is measuring how profinite sets map into X. The sheaf axiom guarantees some coherence among these values. Taking $S = *$ a point, there is an "underlying set" $X(*)$. Scholze (2020)

Consider the following construction of a very familiar topological space as the quotient of a profinite set.

Let T be a compact Hausdorff space. Then a classical and somewhat weird fact is that T admits a surjection $S \to T$ from a profinite set S. One construction is to let S be the Stone-Čech compactification of T^δ, where T^δ is T considered as a discrete set. This lets one recover T as the quotient of S by the equivalence relation $R = S \times_T S \subset S \times S$. Thus, compact Hausdorff spaces can be thought of as quotients of profinite sets by profinite equivalence relations....This is what happens in the condensed perspective, which only records maps from profinite sets. Scholze (2020)

Intuitively, we can think of the "condensation" process as collapsing everything attached to a point onto the point itself; the collapsed representation can then be "fluffed up" into an explicit representation of the collapsed information, when needed, by the application of some operator, in our case the pair $\mathbf{G}, \mathbf{G}^{sh}$.

The étale topology (étale means "slack" or "relaxed") used here is a Grothendieck topology which is defined in positive characteristic, is finer than the Zariski topology, and resembles the Euclidean topology. Informally, étale morphisms are the algebraic equivalent of local diffeomorphisms between manifolds. The pro-étale topology Bhatt & Scholze (2013) resembles the étale topology, but can better handle infinite constructions in cohomology. The pro-étale site contains all finite limits and is locally contractible. Moreover, any site gives rise to a category of sheaves and **Sets** are sheaves on a point.

Now consider the "current" event $\mathbb{V}_c$ located at "the present" regarded as a point. Recall from §3 that a typing C_c of $\mathbb{V}_c$ is just an assigment of subgraphs of the type hierarchy $Type$ to the objects in $\mathbb{V}_c$. This assignment is a finite set of jointly surjective maps, i.e. $C_c : Type \to \mathbb{A}_c$ can also be thought of as a set $\{\xi_i\}$ of maps $\xi_i : Type \to a_i$ for each object $a_i \in \mathbb{A}_c$. We can, therefore, regard $\mathbb{V}_c$ as a pro-étale site on the present; these sites inherit the discrete (indeed Grothendieck[5]) topology of events described earlier.

With this identification of $\mathbb{V}_c$ with $*_{\text{proét}}$ on the present, we can state:

Theorem 3. *An entropic typing is a condensed set.*

Proof. With the above identification, the local functor $\mathbf{F_c} : \mathbb{V}_c \to C_c$ is a presheaf on $*_{\text{proét}}$. Hence all that is required is to extend $\mathbf{F_c}$ by a map $\mathbf{F_c} \to \mathbf{F} \to \widetilde{\mathbf{F}}$, and we have a sheaf over $*_{\text{proét}}$ by Lemma 2. The required extension is, clearly, the adjoint pair $\mathbf{G}, \mathbf{G}^{sh}$ satisfying Eq. (11) and (12). These exist by construction for any entropic typing. □

What Theorem 3 does, intuitively, is just to remove the need for a "target" event from the sheaf-construction constraints given by Eqs. (10) and (11). All memories are, in this case, constructed from the present by FCHs, with no "ground truth" targets as constraints.

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[5]As we began in §2 with an informal notion of the relation between experienced events, we can simply stipulate that their topology is Grothendieck. Nothing we have done is inconsistent with this slightly stronger notion of discreteness.

We can restate Theorem 3 in the language of representable functors, using the standard definition as follows:

Definition 13. *For a locally small category* $\mathbf{C}$*, a presheaf on* $\mathbf{C}$ *or equivalently, a functor* $\mathbf{f} : \mathbf{C^{op}} \rightarrow \mathbf{Set}$ *on the opposite category of* $\mathbf{C}$ *and with values in* $\mathbf{Set}$ *is* representable *if it is naturally isomorphic to a hom-functor:*

$$\mathbf{h_X} := \mathbf{hom_C}(\cdot, X) : \mathbf{C^{op}} \rightarrow \mathbf{Set}$$

that sends an object $U \in \mathbf{C}$ *to the hom-set* $Hom_{\mathbf{C}}(U, X)$ *in* $\mathbf{C}$ *and that sends a morphism* $\alpha : U' \rightarrow U$ *in* $\mathbf{C}$ *to the function which sends each morphism* $U \rightarrow X$ *to the composite* $(U' \overset{\alpha}{\rightarrow} U) \rightarrow X$.

As noted in the discussion of Eq. (12), the existence of $\mathbf{G}, \mathbf{G}^{sh}$ renders time functorial, and hence requires all diagrams with horizonal time arrows and vertical type-assignment arrows to commute. The above conditions are, therefore, satisfied whenever Eq. (11) and (12) are satisfied, i.e. whenever a typing is entropic.

This construction suggests a precise formal model of the implementation of human episodic memory, i.e. of the structure of the memory-encoding "engram" (Eichenbaum, 2016; Josselyn & Tonegawa, 2020) as implemented by networks of neural connections:

Prediction: The (episodic) engram is a representation of $\mathbf{G}^{sh}$.

The observed context-dependence of episodic recall follows immediately from the definition of $\mathbf{G}^{sh}$ in this model. If this prediction is correct, humans reconstruct an experienced time, at each instant of recall, by reconstructing a memory. Hence time itself is condensed to "the present" and reconstructed, as a remembered sequence of events, only when necessary.

This condensed representation of episodic memory, and therefore of event-dependent time, also provides a natural link to the traditional notion of an "objective" continuous time. Pro-étale sites simplify open sets, so condensed sets simplify topological spaces:

Proposition (Proposition 3.1 Clausen & Scholze (2021)): The forgetful functor from the category of topological spaces to condensed sets is a faithful functor. It becomes fully faithful when restricted to compactly generated spaces. This functor admits a left adjoint, which sends a condensed set $\mathcal{T}$ to the topological space given by the underlying set $\mathcal{T}*$ of $\mathcal{T}$ equipped with the quotient topology induced by the map $\underset{S \rightarrow \mathcal{T}}{\sqcup} S \rightarrow \mathcal{T}*$ where S runs over all (κ-small) profinite sets mapping into $\mathcal{T}$. The counit of this adjunction coincides with the counit $X^{cg} \rightarrow X$ of the adjunction between (κ-small) compactly generated spaces and topological spaces.

Hence we can see the discrete time constructed from the present by Theorem 3 as a local, observer-specific coarse-graining of a continuous time. Nothing in the construction, however, guarantees any straightforward relationship between distinct such coarse-grainings. The local "times" of different experiencing agents will, in general, not be commensurable.

11   Assumptions, implications, and proposed experimental tests

Our goal here has been to develop a strict, well-defined formal representation of a previously informal notion, that of constructive episodic memory and hence of constructive, agent-specific, experienced time. Such formal representations serve to make assumptions explicit and to raise new questions to the status of explananda. They also allow rigorous, purely formal assessments of consistency between theories developed from different starting assumptions and intended to model and/or explain different phenomena. The present formalism, for example, provides a model of the agent-specific time reference frame required for full compliance with the Markov blanket condition imposed on agents by the Free-Energy Principle (Fields, Friston, Glazebrook and Levin, 2022).

We assumed in §2 that long experienced times could be treated independently of space. The independence of space as a "container" through which systems evolve in time is fundamental to classical (Newtonian) physics, and is at least approximated by the "folk physics" that humans develop in early childhood (Bjorklund & Ellis, 2014) and appear to largely share with other large-brained mammals. Indeed, it is the intuitive nature of space-time independence that renders relativistic physics counter-intuitive. Recent experimental work indicates that space and time representations are dissociable at the functional-network level in humans for short-duration event sequences (Schonhaut, Aghajan, Kahana & Fried, 2022), suggesting at least that the same is true for the long-time representations of interest here. Such dissociability is assumed by all models that treat spatial representations as "maps" that are invariant across time, as they must be to support flexible composition of sequences of actions either in memories or in plans (McNamee et al., 2022). While the construction of spatial reference frames is obviously of interest, we defer this to future work.

Experienced long times being entropic is a requirement for temporal directionality and hence a past-future distinction. Equation (8) states this requirement, so it or an equivalent is an essential component of any theory of time that supports human phenomenology, or indeed, of any theory consistent with the 2nd Law of Thermodynamics. The sheaf-theoretic treatment developed in §6 - 9 follows from Eq. (8) together with the use of FCHs to maintain object identity across gaps in observation. The formal development in these sections makes precise the empirically-motivated "picture" of FCH construction in Fields (2012). More generally, it provides an explicit formal mechanism for the idea of object

continuity over time that underlies the operational notion of object persistence.

The formal development of §10 likewise provides a precise statement of the informal, empirically-motivated idea of episodic memories being constructive. It predicts, in particular, that episodic engrams encode executable operations – representations of the functor $\mathbf{G}^{sh}$ – that generate particular memories given particular cues. Hence it predicts that retrospective and prospective memory are *mechanistically* symmetrical. This symmetry can be made manifest by formulating local (to the present) analogs of the standard qualitative temporal-logic operators `Always`, `Sometimes`, and `Never`, as well as quantitative extensions such as `At least twice in the past`, etc. Let $R(a, b, \dots)$ be an arbitrary relation on a finite set of objects $\{a, b, \dots\}$. We can define, relative to an event $\mathbb{V}_c$:

- `Always` $(R(a, b, \dots) := \ \forall$ sections s of $\widetilde{\mathbf{F}}$, $(R(a, b, \dots)$ on s.

- `Sometimes` $(R(a, b, \dots) := \ \exists$ a section s of $\widetilde{\mathbf{F}}$ such that $(R(a, b, \dots)$ on s.

- `Never` $(R(a, b, \dots) := \ \neg\exists$ a section s of $\widetilde{\mathbf{F}}$ such that $(R(a, b, \dots)$ on s.

Alternatively, $(R(a, b, \dots)$ on all, some, or no images of $\mathbf{G}^{sh}$ acting on $\mathbf{F}_c$. Note how these definitions depend on time being entropic, i.e. on $\mathbb{V}_c$ encoding "past" relational information that applies equally to constrain the "future" of prospective memory. Such information can clearly be considered to include probability distributions, i.e. $\mathbb{V}_c$ can be taken to encode prior probabilities of predictable events.

The symmetry of $\mathbf{G}^{sh}$ raises an obvious question: the theory requires that time be entropic, but what guarantees this operationally? What assures, at the implementation level, that past relational information is built into current representations, as required by Eq. (8)? This is an empirical question for which we can make a testable prediction:

Prediction: Episodic memory depends mechanistically on stigmergic memory.

As pointed out earlier, episodic memory depends on the persistence of typings, and hence on semantic memory. This prediction is stronger: episodic memory depends also on the existence of external records, accessible in the present, that encode past relational information. Semantic memory of the existence of such records may substitute in part for current access to stigmergic records, but not entirely. Hence we predict a fundamental asymmetry in the implementation of memory:

Prediction: Stigmergic memory is primary; semantic and episodic memory are derivative.

This prediction can be empirically tested in basal cognitive systems, and appears to be borne out: stigmergic memories are ubiquitous across phylogeny, while semantic and episodic memories appear to be limited to late-evolved, large-brained vertebrates and

cephalopods (Fields, Glazebrook & Levin, 2021). It is also supported by considerable observational evidence in humans. For humans, a major source of stigmergic memories is other humans; written records, photographs, other artifacts, landscape modifications, etc. provide additional sources. Both the content and the credibility of these sources are highly manipulable. False-memory induction in therapeutic and other settings (e.g. Loftus, 2017) thus provide evidence for the role of stigmergic memory in the construction of episodic memories. Induced biases in planning provide similar evidence in the prospective direction. We can, indeed, expect radical manipulations of stigmergic memory to phenocopy organic neurological dysfunction or neurodegeneration, as plausibly observed in cases of delusional-belief induction by cults, conspiracy theories, and psychological torture.

While both stigmergic and semantic memories often encode explicit, external-clock referenced time information, the model developed here implies, as noted earlier, that such external time information is insufficient for an experience of time. This dissociation between an impersonally known and a personally experienced past and, analogously, between an impersonally predictable and a personally plannable future have been observed in amnesic patients (Klein, Loftus & Kihlstrom, 2002).

The symmetry of $\mathbf{G}^{sh}$ also implies that both reconsolidation and competition effects observed in memory encoding (reviewed in Josselyn & Tonegawa, 2020) will also be observable in planning, where they will present as context-specific biases. More precisely, we can predict that:

Prediction: Episodic recall and planning of an event $\mathbb{V}$ employ the same engram complex.

Activation of "nearby" memories $\mathbb{V}_i$ of a target event $\mathbb{V}_j$ should disrupt planning of $\mathbb{V}_j$.

Finally, the symmetry of $\mathbf{G}^{sh}$ implies that the source-monitoring processes that distinguish memories from plans and hence support a phenomenological past/future distinction are independent of the engram itself. As in the case of perception versus imagination source monitoring (e.g. Dijkstra Kok & Fleming, 2022), we can expect memory versus planning source monitoring to be a metacognitive process that synthesizes contextual evidence, including strength of top-down control. We predict that plans and memories can be confused in contexts in which top-down control is sufficiently relaxed.

12   Conclusions and extensions

In this paper, we have developed the basic formalism needed to represent events and episodic memory in a localized, agent-dependent functorial time. This formalism makes precise the idea that episodic memories are constructs enacted in the present, and frees temporal reasoning from the ontological constraint of a continuous "objective" time equally shared by all experiencing agents. We are led quite naturally to a significant empirical prediction, that episodic memories are encoded as representations of a particular functor,

$\mathbf{G}^{sh}$, that constructs retrospective pasts and prospective futures subject to the constraint of time being entropic, i.e. satisfying Eq. (8).

The constructions we report here leave open the question of how to most effectively represent typing inconsistencies in either retrospective pasts or prospective futures. Planning, in particular, typically involves projecting multiple, mutually-inconsistent future environmental contingencies and courses of action. Retrospective memory can, however, also involve uncertainties about what actually happened, particularly during periods of non-observation.

One approach to these questions is suggested by Fig. 4 and the formalism of topological field theories: it is to reformulate episodic memory in terms of cobordisms, and to allow topologically-complex evolutions that involve multiple intermediate boundaries. A second, more algebraic approach is suggested by Theorem 2: it is to generalize in the direction of derived categories and perverse sheaves. These approaches may, indeed, prove to be closely related. Working from the "diamond" construct of Scholze (2017), one of us recently conjectured a pro-diamond (Dobson, 2021a) towards a theory of pro-emergent time. This construction naturally suggests a holographic interpretation (Dobson, 2021b) and hence a formulation in terms of cobordisms.

To conclude, we suggest that viewing time as event-dependent, constructive, and indeed as condensed into a functorial operation on the present opens new opportunities for modeling episodic memory, planning, and temporal reasoning. The formal methods enabling such a view have intriguing connections to field theories, particularly topological field theories. They enable novel predictions that are potentially testable as methods for analyzing neuronal networks in humans and other organisms are further developed. Finally, such methods suggest a deeper connection between models of time and constructive, experience- and reference-frame dependent models of space.

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